International Journal of Applied Sciences: Current and Future Research Trends
(IJASCFRT)
ISSN (Print) , ISSN (Online)

https://ijascfrtjournal.isrra.org/index.php/Applied_Sciences_Journal

The System of Quantum Properties of Superposition for Lambda Type Three-level Lasers

Takele Teshome Somano*

Email: takeleteshome36@gmail.com

Abstract

The squeezing features, as well as the built-in light bulbs figure produced by the quantum superposition system to prepare a lambda type of lasers for three levels have been adjusted. We determined what the quadrature variation and photon number of the pit route numbers mean with the help of Langevin c-number mathematical solutions corresponding to the standard order. The results show that the light produced by the process used is in a state of compression. We have made our analysis of the conditions of simple pressure can reveal a system that is considered under the condition that the total continuous damage is greater than the benefit and pressure that occurs in minus-quadrature. In addition, we receive the help of Q-function and operator to strengthen the size of the surface, and the fixed beams are determined by the quadrature variation and the number of photon definitions. The result shows that the lower the photon value and the quadrature variation of the highly illuminated beam the total number of the mean photon number and the quadrature variation of the illuminated beam.

Keywords: Superposition; Quantum quadrature; Photon Statistics.

1. Introduction

Quantum machines, which define the performance of reliance and energy on a small body scale, are the most appropriate natural system ever built. Many of its predictions can be displayed in ten decimal places or more. Its concepts provide an explanation of many conditions from the massive Periodic Table of Elements to the normal operation of lasers, microchips, diode-emitting diodes and MRI clinical scanning. The ability of the carpet to see and manage objects on the quantum stage is most of the highest aspirations of 20th century science.

----

* Corresponding author.

It is important to develop physics, chemistry, digital engineering and nanotechnology, develop new computer systems for unconventional energy, and discover how atoms behave in a large array, including crystals of semiconductors or metal-oxide layers of superconductors. However, the most successful researchers in JQI and elsewhere have begun to manipulate and resort to extremes, which can be difficult to control for many reasons. Quantum optics offers mainly quantum residential environments made up of multiple optical structures including lasers and the effects of atomic energy. The interaction between radiation and dependence (atoms) lies in the heart of quantum optics. There was a great deal of fun within the inspection of the meeting places and the gentle figures made with three lasers [1 - 17]. A 3-stage laser can be described as a quantum optical device in which three-stage atoms are first arranged in a corresponding 2-dimensional space, embedded in an empty space connected to a vacuum pool by a single port mirror. The three-stage device incorporates dipole transients. 3 styles of 3 stage buildings, all taking their names from the Greek and Latin characters are the Lambda device have dropped distances combined with fun, the Cascade device has a happy world, connected to the middle world, and one place connected to the lower world, and the Vee device (V) , with one world united by 2 happy worlds. The 3-stage laser incorporates a blank space where three atoms-stage are placed at a fixed charge and removed from the empty space after a certain time τ T. Three distances are represented by | a>, | b>, and | c>. Pressure is one of the most unusual skills to attract the first level of entertainment. At low pressures, the sound in a single quadrature is below the parallel or empty world stage at the price of large fluctuations within a different quadrature created by uncertainty within the quadrature pleasurable relationship [1, 3-13]. Thin fingers have the ability to use skill with low voice exchange and signal acquisition that may be easier. A 3-phase laser can be described as a quantum optical device in which three-stage atoms, starting with a series of corresponding propulsions of the 2nd phase, are placed in a vacuum. When a three-stage atom makes a transition from the top to the lowest point, photons are formed. The combination of the 2nd atoms of the atom is charged with the old unpleasant properties of soft generated. Normally, an atomic junction can be drawn from a phase 3 atom by attaching the corresponding gentleness or by preparing the atom to begin with a stiffness that does not match those distances. There are many quantum optical structures that can generate gentleness with unusual skills including squeezing, grip, and combat etc. We rely on empty space modes to match the 2 | version a> ↔ | b> no | a> ↔ | b> whether the dipole is allowed with a direct switch between the stage | a> → | | c> to be a declined deposit. It is assumed that the case in which the atoms will first be arranged in the Superposition of | a> no | c>. When a 3-phase cascade atom decomposes from the top to reduce the distances the photons emit. If 2 photons have equal frequencies, then a 3-phase atom is known as a decaying phase 3 atoms, in any case more commonly known as a negative phase 3 atom. It has been found that two photons produced with a cascade 3-stage laser device have compressed housings under precise conditions due to the interaction between the photons.

2. Three-level atom with radiation

In this chapter we seek to drive the Hamiltonian describing the interaction of a radiation with lambda type three-level atom and the master equation for three-level laser coupled to vacuum reservoir.

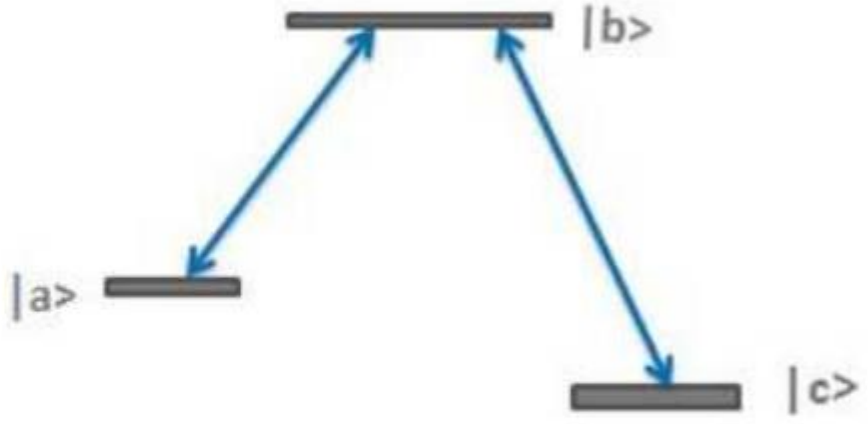

Figure 1: Schematic representation of a three-level atom in a lambda Configuration.

2.1. The Hamiltonian

In this section we seek to drive the Hamiltonian describing the interaction of a radiation with three-level atom. The interaction of a one-electron atom having mass *m* and charge e with a single-mode radiation represented by the vector potential *A*, is described by the Hamiltonian

$$\hat{H}_1 = \frac{e}{m}\hat{p}\,.\hat{A} \,+\, \frac{e^2}{2m}\hat{A}^2 \qquad (1)$$

Where $\hat{P}$ is the canonical momentum. Since, we can neglect the second term in

Eq.(2.1). Hence the interaction Hamiltonian can be put in the form

$$\hat{H}_1 = \frac{e}{m}\hat{p}\,.\hat{A} \qquad (2)$$

It proves to be more convenient to express this Hamiltonian in terms of electric field operator. To this end, we note that

$$\frac{d}{dt}(\hat{r}.\hat{A}) = -\frac{ie}{m\hbar}\left[\hat{r}.\hat{A}\,,\hat{p}\,.\hat{A}\right] \qquad (3)$$

n which $\hat{r}$ is the position operator for the electron. Taking into account the fact that the vector potential commute with the position and momentum operators, one readily finds

$$\left[\hat{r}.\hat{A}\,,\hat{p}\,.\hat{A}\right]= i\hbar\hat{A}^2. \qquad (4)$$

It then follows that

$$\frac{d}{dt}(\hat{r}.\hat{A}) = \frac{e}{m}\hat{A}^2 \qquad (5)$$

On the other hand, we have

$$\frac{d}{dt}(\hat{r}.\hat{A}) = \hat{A}.\ \frac{d\hat{r}}{dt} + \hat{r}.\frac{d\hat{A}}{dt}, \qquad (6)$$

so that combination of Eqn. (2.5) and (2.6) leads to

$$\hat{A}.\frac{d\hat{r}}{dt} = \frac{e}{m}\hat{A}^2 - \hat{r}.\frac{d\hat{A}}{dt} \qquad (7)$$

Furthermore, with the aid of the relation

$$\hat{p} = \mathrm{m}\frac{d\hat{r}}{dt} - \mathrm{e}\hat{A}, \qquad (8)$$

One can write

$$\hat{p} \cdot \hat{A} = \mathrm{m\ d}\frac{d\hat{r}}{dt} \cdot \hat{A} - \mathrm{e}\hat{A}^{2.} \qquad (9)$$

Now on account of Eqn. (2.7) and (2.9), we have

$$\hat{p} \cdot \hat{A} = -\mathrm{m}\hat{r} \cdot \frac{d\hat{A}}{dt}. \qquad (10)$$

In view of this the interaction the Hamiltonian described by Eqn. (2) can therefore be

Put in the form

$$\hat{H}_{\mathrm{I}} = -\mathrm{e}\hat{r} \cdot \frac{d\hat{A}}{dt}. \qquad (11)$$

It can be easily established that

$$\frac{d\hat{A}}{dt} = \frac{\partial\hat{A}}{\partial t} + (-\vec{v} \cdot \nabla)\hat{A} \qquad (12)$$

A simpler form of Eq. (2.12) can be obtained by applying the electric dipole approximation

$$e^{ik.r} = 1 \qquad (13)$$

2.2. Quantum approximation

This is evidently justified d provided that *k.r* << 1. Inspection of the vector potential in the electric dip ole approximation, on the position c o ordinates , it then turns out in this approximation that

$$(-\vec{v}\cdot\nabla)\hat{A}=0\,, \tag{14}$$

and hence in the Coulomb gauge we have

$$\frac{d\hat{A}}{dt}=\frac{\partial\hat{A}}{\partial t}=-\hat{E}. \tag{15}$$

In view of this result, we see that

$$\hat{H}_{\mathrm{I}}=e\hat{r}(t)\cdot\hat{E}\,(x\,,t). \tag{16}$$

The electric field operator for two-mode light in the electric dipole approximation has the form

$$\hat{E}(x,t)=ie\left\{\,[\frac{\hbar\omega_a}{2\mathcal{E}_{oV}}]^{\frac{1}{2}}\,(\hat{a}\,e^{-i\omega_a t}-\hat{a}^{+}\,e^{i\omega_a t}\,)+\left\{[\frac{\hbar\omega_b}{2\mathcal{E}_{oV}}]^{\frac{1}{2}}\,\left(\hat{b}\,e^{-i\omega_b t}-\hat{b}^{+}\,e^{i\omega_b t}\,\right)\right\}\mathbf{u}. \tag{17}$$

And

$$\hat{r}(t)=\,r_{ab}\,(\hat{\sigma}_{ab}\,e^{i\omega_a' t}+\hat{\sigma}_{ba}\,e^{-i\omega_a' t})+r_{bc}\,(\hat{\sigma}_{bc}\,e^{i\omega_b' t}+\hat{\sigma}_{cb}\,e^{-i\omega_b' t}). \tag{18}$$

The result is

$$\hat{H}_{\mathrm{I}}=i\hbar g\left[\left(\hat{\sigma}_{ab}\,\hat{a}e^{-i(\omega_a-\omega_a')t}-\hat{\sigma}_{ba}\,{}_{\hat{a}^{+}}e^{-i(\omega_a-\omega_a')t}\right)+\left(\hat{\sigma}_{bc\,\hat{b}}e^{i(\omega_b-\omega_b')t}+\hat{\sigma}_{cb}\,\hat{b}^{+}e^{-i(\omega_b-\omega_b')t}\right)\right]. \tag{19}$$

3. Master equation

In this section we determine the master equation for lambda type three-level laser in two-mode vacuum reservoir

3.1. Lambda- type three-level laser

We consider three-level lasers in which three-level atoms in Λ configuration are injected at a constant rate ra and removed from the cavity after a certain time τ. We denote the top and the two bottom levels of three-level atom by | b > , | a > and | c > as shown in Fig 2.1 we assume that the cavity modes to be at resonance with the two transitions | b > ↔ | a > and | b > ↔ | c > , are dipole allowed and direct transition between level | a > ↔ | c > to be dipole forbidden. The interaction of Λ three-level atom with the cavity modes can be described in the interaction picture by the Hamiltonian

$$\hat{H}_{\mathrm{I}}=ig\left[(\hat{\sigma}_{ab}\,\hat{a}-\hat{\sigma}_{ba}\,{}_{\hat{a}^{+}})+\left(\hat{\sigma}_{bc\,\hat{b}}+\hat{\sigma}_{cb}\,\hat{b}^{+}\right)\right] \tag{20}$$

We take the initial state of a three-level atom to be

$$| \psi A(0) > = Ca(0)| a > + Cc(0)|c > \quad (21)$$

The master equation for Λ three-level laser coupled to a two-mode vacuum reservoir can be

Written as

$$\frac{d}{dt}\hat{\rho} = \frac{1}{2}\left(\kappa + B\rho_{aa}^{(0)}\right)(2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}) + \frac{1}{2}\left(\kappa + B\rho_{cc}^{(0)}\right)\left(2\hat{b}\hat{\rho}\hat{b}^{+} - \hat{b}^{+}\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^{+}\hat{b}\right) + \frac{1}{2}\left(B\rho_{ac}^{(0)}\right)\left(2\hat{a}\hat{\rho}\hat{b}^{+} - \hat{\rho}\hat{b}^{+}\hat{a} - \hat{b}^{+}\hat{a}\hat{\rho}\right) + \frac{1}{2}\left(B\rho_{ac}^{(0)}\right)\left(2\hat{b}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{b}\hat{\rho} - \hat{\rho}\,\hat{a}^{+}\hat{b}\right) \quad (22)$$

3.2. C-number langavin equations

The c- number Langavin equation associated to the normal orders.

$$\frac{d}{dt} < \alpha > = -\frac{1}{2}\,\mu_a < \alpha > -\frac{1}{2}\,v < \beta > \quad (23)$$

$$\frac{d}{dt} < \beta > = -\frac{1}{2}\,\mu_c < \beta > -\frac{1}{2}\,v < \alpha > \quad (24)$$

$$\frac{d}{dt} < \alpha^2 > = -\mu_a < \alpha^2 > - v < \alpha\beta > \quad (25)$$

$$\frac{d}{dt} < \beta^2 > = -\mu_c < \beta^2 > + v < \alpha\beta > \quad (26)$$

$$\frac{d}{dt} < \alpha^*\alpha > = -\mu_a < \alpha^*\alpha > -\frac{1}{2}\,v(< \alpha^*\beta > + < \beta^*\alpha >) \quad (27)$$

$$\frac{d}{dt} < \beta^*\beta > = -\mu_c < \beta^* - \frac{1}{2}\,(\mu < \alpha^*\beta + v\beta^*\alpha >) \quad (28)$$

$$\frac{d}{dt} < \alpha^*\beta > = -\frac{1}{2}(\mu_a + \mu_c) < \alpha^*\beta > -\frac{1}{2}\,v(< \alpha^*\beta > + < \beta^*\alpha >) \quad (29)$$

$$\frac{d}{dt} < \alpha\beta > = -\frac{1}{2}(\mu_a + \mu_c) < \alpha\beta > -\frac{1}{2}\,v(< \beta^2 > + < \alpha^2 >) \quad (30)$$

On the base of this equation Eqn (23) and (24)]

$$\frac{d}{dt}\,\alpha(t) = -\frac{1}{2}\,\mu_a\alpha(t) - \frac{1}{2}\,v\beta(t) + f_\alpha(t) \quad (31)$$

$$\frac{d}{dt}\,\beta(t) = -\frac{1}{2}\,\mu_c\beta(t) - \frac{1}{2}\,v\alpha(t) + f_\beta(t) \quad (32)$$

Where $f_\alpha(t)$ and $f_\beta(t)$ are noise forces. The solution of equation (31) and (32) can be written as

$$\alpha(t) = \alpha(0)e^{\frac{-\mu_a t}{2}} - \int_0^t dt' e^{\frac{-\mu_a(t-t')}{2}} \quad (\frac{1}{2}v\,\beta(t') - f_\beta(t')) \tag{33}$$

$$\beta(t) = \beta(0)e^{\frac{-\mu_c t}{2}} - \int_0^t dt' e^{\frac{-\mu_c(t-t')}{2}} \quad (\frac{1}{2}v\,\alpha(t') - f_\beta(t')) \tag{34}$$

The solution of coupled differential equation Eqn. (33) and Eqn. (34) can be written in the matric form as

$$\frac{d}{dt}Y(t) = \frac{1}{2}MY(t) + F(t) \tag{35}$$

$$Y(t) = \begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} \tag{36}$$

$$M = \begin{pmatrix}\mu_a & v\\ v & \mu_c\end{pmatrix} \tag{37}$$

$$F(t) = \begin{pmatrix}f_\alpha(t)\\ f_\beta(t)\end{pmatrix} \tag{38}$$

We next proceed to find the eigenvalues and eigenvectors of the matrix M. Applying the eigenvalue equation

$MU_i = \lambda U_i$, We find the characteristic equation

$$\lambda^2 - \lambda(\mu_a + \mu_c) + \mu_c\mu_a - v^2 = 0$$

We finally obtain (39)

$$\alpha(t) = p_1(t)\,\alpha(0) + q_1(t)\beta(0) + H_1(t) \tag{40}$$

$$\beta(t) = p_2(t)\,\beta(0) + q_2(t)\alpha(0) + H_2(t) \tag{41}$$

Where

$$H_1(t) = \int_0^t p_1(t-t')f_\alpha(t') + q_1(t - t')f_\beta(t')\,dt' \tag{42}$$

$$H_2(t) = \int_0^t p_2(t-t')f_\beta(t') + q_2(t-t')f_\alpha(t')\,dt' \tag{43}$$

3.3. The q –function

We now proceed to determine the Q function for the cavity mode produced by a Λ three level lasers coupled to vacuum reservoir. The Q function for a two-mode light is expressible as

$$Q(\alpha,\beta,t) = \frac{1}{\pi^2}\int \frac{d^2z}{\pi}\frac{d^2w}{\pi} \quad \Phi(z,w,t)e^{z*\alpha - z\alpha* + w*\beta - w\beta*} \tag{44}$$

where

$$\Phi\,(z,\,w,\,t) = T\,r(\hat{\rho}e^{-z*\hat{a}(t)}e^{z\hat{a}^+(t)}e^{-w*\hat{b}(t)}e^{w\hat{b}^+(t)}$$

We can write in terms of c-number variables associated to the normal ordering as

$$\Phi\,(z,\,w,\,t) = e^{-z*z}e^{-w*w} < e^{z\alpha*(t) - z*\alpha(t) + w\beta*(t) - w*\beta(t)} > \tag{45}$$

We note that $\alpha\,(t)$ and $\beta\,(t)$ are Gaussian variables with a vanishing mean.

In the view of this finally Q-function for lambda three-level lasers,

$$Q(\alpha,\beta) = \frac{1}{\pi^2}\exp(-\alpha*\alpha - \beta*\beta) \tag{46}$$

4. Quadrature squeezing

Here we seek to determine the quadrature variance s for a superposed two- mode light beams produced by Λ-type three- level lasers. We define the quadrature variance for a superpose d two- mode light beams by

$$\Delta c^2{}_{\pm} = < \hat{c}_{\pm}, \hat{c}_{\pm} > \tag{47}$$

Where

$$\hat{c}_{\pm} = \sqrt{\pm(\,\hat{c}^+, \hat{c})}, \tag{48}$$

are the plus and minus quadrature operators for the superposed two-mode light beams, $\hat{c}$ and $\hat{c}^+$ are the annihilation and creation operators for the superposed two-mode light.

$$[c_+\,, c_-] = 8i. \tag{49}$$

In view of the relation in above equation, the superposed two-mode light is said to be in squeezed state if $either\ \Delta c - < 4\ or\ \Delta c + < 4\ such\ that\ \Delta c\ + \Delta c - \geq 4$. with the aid of above equation,

We get

$$\Delta c^2{}_{\pm} = 4 + 2 < \hat{c}^+\hat{c} > \pm < \hat{c}^{+2} > \pm < \hat{c}^2 > - 2 < \hat{c}^+ >< \hat{c} > \mp < \hat{c}^+ >^2 \mp < \hat{c} >^2. \tag{50}$$

Taking to account Eqn.(5.45) and $< \hat{c}^+ > = < \hat{c} >$, we have

$$\Delta c^2{}_{\pm} = 4 + 2 < \hat{c}^{+}\hat{c} > \pm 2 < \hat{c}^2 > \qquad (51)$$

Using the density operator, we write as and c- Langavin equation

$$\Delta c^2{}_{\pm} = 4 \ + A(1-\eta)\left[\frac{2\kappa(4\kappa+2A\eta+A)-A^2(1-\eta)}{2\kappa(2\kappa=A\eta)(\kappa+A\eta)}\right] + A\sqrt{1-\eta^2}\left[\frac{2\kappa(4\kappa+3A\eta+A)-A^2(1-\eta^2)}{2\kappa(2\kappa=A\eta)(\kappa+A\eta)}\right] \qquad (52)$$

The equation represents the quadrature variances for superposition of the light beams produced by lambda three-level lasers. Fig .2 represents the variances of the minus quadrature [Eqn. (50)] versus η for different values of A. This figure indicates that the degree of squeezing increases with the linear gain coefficient and almost perfect squeezing can be obtained for large values of the linear gain coefficient and for small values of η. Moreover, the minimum value of the quadrature variance described by Eqn. (52) for A = 1, κ = 0.8, is found to be Δc_2^{-} = 2.79 and occurs at η = 0.35 this result implies that the maximum intracavity squeezing for the above values is 30.25 percent below the coherent-state level.

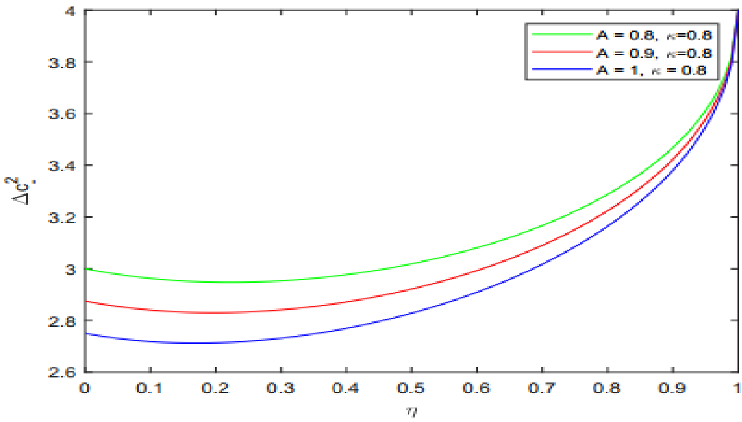

Figure 2: Plots of the minus quadrature variance above equation versus η for κ = 0.8, and for different values of the line again coefficient.

5. Photon statistics

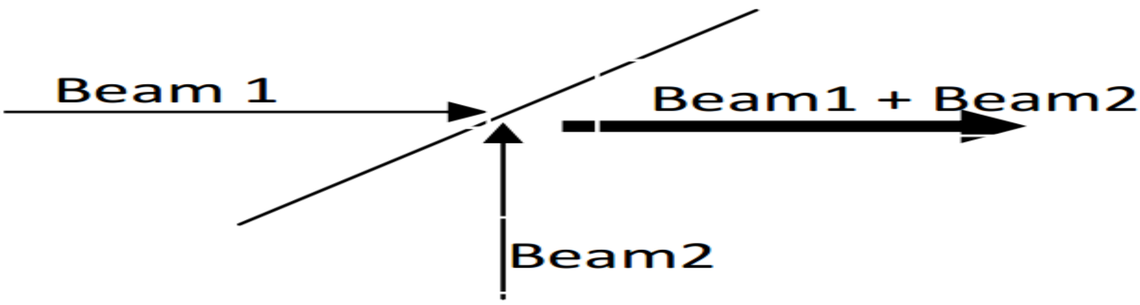

Figure 3: Schematic representation of the superposition of two- mode light beams of lambda- type three-level lasers.

In this chapter we wish to study the statistical properties of a superposition of a two-mode light beams produced by lambda-type three- level lasers employing the individual beams' Q- functions and the density operator for the superposed beams .We consider the two light beams from perpendicular direction are propagating into a mirror having one side totally transmitting and the other side totally reflective as seen in Fig. (2).

5.1. Density operator

Suppose ˆρ (t) be the density operator for the superposition of two beams. We can write ρˆ (t) in the normal order as

$$\hat{\rho}(t) = \sum_{ijkl} c_{ijkl}(t)\, \hat{a}^{+i}\hat{a}^{j}\hat{b}^{+k}\hat{a}^{l} \quad (53)$$

$\hat{I} = \int \frac{d^2\alpha_1}{\pi}\frac{d^2\beta_1}{\pi}|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|$, and the relation

$$|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|\, \hat{a}^{+i}\hat{b}^{+k} = \alpha_1^{*i}\beta_1^{*l}|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|, \quad (54)$$

$$|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|\, \hat{a}^{+i}\hat{b}^{+k} = (\alpha_1 + \frac{\partial}{\partial\alpha_1^*})^j(\beta_1 + \frac{\partial}{\partial\beta_1^*})^k|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|$$

$$\hat{\rho} = \int d\alpha_1 d\,\beta_1 Q_1\left(\alpha_1^*, \beta_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}; \beta_1 + \frac{\partial}{\partial\beta_1^*}\right)\hat{D}\,(\alpha_1, \beta_1)\hat{\rho}_o\,\hat{D}^+(-\alpha_1 - \beta_1\,) \quad (55)$$

Where

$$Q_1 = \left(\alpha_1^*, \beta_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}; \beta_1 + \frac{\partial}{\partial\beta_1^*}\right) = \frac{1}{2\pi}\sum_{ijkl} c_{ijkl}(t)\, \alpha_1^{*i}\beta_1^{*l}(\alpha_1 + \frac{\partial}{\partial\alpha_1^*})^j(\beta_1 + \frac{\partial}{\partial\beta_1^*})^k \quad (56)$$

$\hat{D}\,(\alpha_1, \beta_1) = e^{\alpha_1\hat{a}^+ + \beta_1\hat{b}^+ - \alpha_1^*\hat{a} - \beta_1^*\hat{b},}$

And $\hat{\rho}_o$ is the density operator for the system at initial time

Assuming that the density operator at initial time is to be in some other state and following the same procedure to obtain the expression in Eq. (5.5), we readily find that

$$\hat{\rho}_o = \int d^2\alpha_2 d^2\beta_2 Q_2\left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}; \beta_2 + \frac{\partial}{\partial\beta_2^*}\right)$$

$$\hat{D}\,(\alpha_2, \beta_2)|0,0\rangle\langle 0,0|\hat{D}^+(\alpha_2, \beta_2\,) \quad (57)$$

In which

$$Q_2 = \left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}; \beta_2 + \frac{\partial}{\partial\beta_2^*}\right)$$

$$=\frac{1}{2\pi}\sum_{pqrs} c_{pqrs}(t)\, \alpha_2^{*p}\beta_2^{*r}(\alpha_2 + \frac{\partial}{\partial\alpha_2^*})^q(\beta_2 + \frac{\partial}{\partial\beta_2^*})^s \qquad (58)$$

$$\hat{D}\,(\alpha_2, \beta_2) = e^{\alpha_2\hat{a}^+ + \beta_2\hat{b}^+ - \alpha_1^*\hat{a} - \beta_2^*\hat{b},}$$

With the aid of Eqn. (5.5) and (5.8) as well as the relation

$$\hat{D}\,(\alpha_2, \beta_2)\hat{D}\,(\alpha_1, \beta_1)|0,0\rangle\langle 0,0|\hat{D}^+(\alpha_1, \beta_1\,)\hat{D}^+(\alpha_2, \beta_2\,) = \;|\alpha_1 + \alpha_2 + \beta_1 + \beta_2\rangle\langle\beta_1 + \beta_2 + \alpha_1 + \alpha_2| \qquad (59)$$

The expectation value of the annihilation operator for two-mode superposed light beams represented by $\hat{}c$ in terms of density operator is given by

$$\langle\hat{a}\rangle = Tr(\hat{\rho}(t)\hat{c}(0)) \qquad (60)$$

Taking account Eqn. () and$\int d^2\alpha_i d^2\beta_i Q_i\left(\alpha_i^*, \beta_i^*, \alpha_i + \frac{\partial}{\partial\alpha_i^*}; \beta_i + \frac{\partial}{\partial\beta_i^*}\right) = 1$, we have

$$\langle\hat{c}\rangle = \langle\hat{a}_1\rangle + \langle\hat{b}_1\rangle + \langle\hat{a}_2\rangle + \langle\hat{b}_2\rangle \qquad (61)$$

$$\langle\hat{a}_i\rangle = \int d^2\alpha_i d^2\beta_i Q_i\left(\alpha_i^*, \beta_i^*, \alpha_i + \frac{\partial}{\partial\alpha_i^*}; \beta_i + \frac{\partial}{\partial\beta_i^*}\right)\alpha_i \qquad (62)$$

$$\langle\hat{b}_i\rangle = \int d^2\alpha_i d^2\beta_i Q_i\left(\alpha_i^*, \beta_i^*, \alpha_i + \frac{\partial}{\partial\alpha_i^*}; \beta_i + \frac{\partial}{\partial\beta_i^*}\right)\beta_i \qquad (63)$$

$with\;\; i = 1{,}2$. Based on the result given by Eqn. (5.12), one can write the operator representing the superposed light beams as

$$\hat{c} = \hat{a}_1 + \hat{a}_2 + \hat{b}_1 + \hat{b}_2 \qquad (64)$$

With the commutation relation

$$[\hat{c}\,, \hat{c}^+] = 4. \qquad (65)$$

We note that the Q-function for a single mode can obtained using the relation

$$Q_i(\alpha_i^*, \alpha_i) == \int d^2\beta_i Q_i(\,\alpha_i, \beta_i, t) \qquad (66)$$

$$Q_i(\alpha_i^*, \alpha_i + \frac{\partial}{\partial\alpha_i^*}) \;=\; Q_i(\alpha_i^*, \alpha_i, t)e^{-Ua_i\alpha_i^*\frac{\partial}{\alpha_i^*}}, \qquad (67)$$

$$Q_i(\beta_i^*, + \frac{\partial}{\partial\beta_i^*})\beta_i \;=\; Q_i(\beta_i^*, \beta_i, t)e^{-Ua_i\beta_i^*\frac{\partial}{\beta_i^*}} \qquad (68)$$

5.2. Mean photon number

In this section we seek to calculate the mean photon number f or superposed two- mode light beams produced by cascade and lambda- three -level lasers. The mean photon number for the superposition of two-mode light beams can be expressed in terms of the density operator as

$$\bar{n} = \langle \hat{c}^{+}\hat{c}\rangle = Tr(\hat{\rho}\hat{c}^{+}\hat{c}) \quad (69)$$

$$\hat{C}^{+}(t)\hat{c}(t) = \int d^2\alpha_1 d^2\alpha_2 d^2\beta_1 d^2\beta_2 \mathrm{Q}_1\left(\alpha_1^*, \beta_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}; \beta_1 + \frac{\partial}{\partial\beta_1^*}\right) \left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}; \beta_2 + \frac{\partial}{\partial\beta_2^*}\right)|\alpha_1 + \alpha_2 + \beta_1 + \beta_2|^2 \quad (70)$$

This equation can be put in the form

$$\hat{C}^{+}(t)\hat{c}(t) = \langle a_1^{+}\hat{a}_1\rangle + \langle a_2^{+}\hat{a}_2\rangle + \langle \hat{b}_1^{+}\hat{b}_1\rangle + \langle \hat{b}_2^{+}\hat{b}_2\rangle + \langle a_1^{+}\hat{b}_1\rangle + \langle \hat{b}_1^{+}\hat{a}_1\rangle$$

$$+\langle \hat{b}_2^{+}\hat{a}_2\rangle + \langle \hat{a}_2^{+}\hat{b}_2\rangle + \langle \hat{b}_2^{+}\hat{a}_2\rangle + \langle \hat{a}_1^{+}\rangle\langle \hat{a}_2\rangle + \langle \hat{a}_2^{+}\rangle\langle \hat{a}_1\rangle + \langle \hat{a}_1^{+}\rangle\langle \hat{b}_2\rangle$$

$$+\langle \hat{b}_2^{+}\rangle\langle \hat{a}_1\rangle + \langle \hat{b}_1^{+}\rangle\langle \hat{a}_2\rangle + \langle \hat{a}_2^{+}\rangle\langle \hat{b}_1\rangle + \langle \hat{b}_1^{+}\rangle\langle \hat{b}_2\rangle + \langle \hat{b}_2^{+}\rangle\langle \hat{b}_1\rangle \quad (71)$$

Carrying out the integration and then performing the differentiation, we find

$$\langle \hat{a}_i^{+}\hat{a}_i\rangle = a_1 - 1 \quad (72)$$

$$\langle a_1^{+}\hat{a}_1\rangle = a_1 - 1 \quad \langle a_2^{+}\hat{a}_2\rangle = 0 \quad (73)$$

Similarly

$$\langle \hat{b}_i^{+}\hat{b}_i\rangle = b_1 - 1 \quad (74)$$

$$\langle \hat{b}_1^{+}\hat{b}_1\rangle = b_1 - 1, \langle \hat{b}_2^{+}\hat{b}_2\rangle = o \quad (75)$$

$$\langle \hat{a}_i\rangle = \langle \hat{b}_i\rangle = \langle \hat{a}_i^{+}\hat{b}_i\rangle = 0 \quad (76)$$

Then finally

$$\hat{C}^{+}(t)\hat{c}(t) = \sum_1^2 \langle \hat{a}_i^{+}\hat{a}_i\rangle + \langle \hat{b}_i^{+}\hat{b}_i\rangle \quad (78)$$

Eqn. (78) represents the mean photon number for the superposition of the cascade and lambda three-level lasers. We observe from Eqn. (78) that the mean photon number for superposition light beams is the sum of the mean photon number of the mean photon number of lambda –type three-level laser.

6. Conclusion

In this study we have seen the squeezing and statistical properties of the light generate d by three- level laser whose cavity modes are coupled to vacuum reservoir. In which the three-level atoms in a Λ configuration and initially prepared in the superposition of the top and bottom levels are injected into a cavity coupled to vacuum reservoir via a single p ort-mirror. Applying the linear and adiabatic approximation scheme we found the master equation for a light produce d by three -level laser from which we obtained the c -number Langevin equations and their solutions. Employing these solutions we found the ant normally ordered characteristic function which was used to find the Q-function of a light beam generated by three -level laser in Λ. High on Q-function we calculate the number of photon number and the quadrature variation In addition; we calculate the Q function of adding more than Λ -type three-level laser beams. We use this function to calculate the congestion operator, the mean photon value and the quadrature variation of the stable condition. We have seen that the rate of increasing pressure s with a direct gain works well for small amounts of η and the almost completely accessible pressure can be obtained by large values of a straight line coefficient. Finally our results show that it is possible to get more pressure occurring in the cascade than superpose d, but the quadrature variation of Λ remains the same.

Acknowledgments

First, I would like to gratitude my God who crucified and sacrificed on the cross to heal me and win my death.

Bibliography

[1]. Fesseha Kassahun, Foundations of Quantum Optics (Lulu Press Inc., North Carolina, 2008).

[2]. Marlan O.Scully and Mr. Suhail Zubairy, Quantum Optics (Cambridge University, 1997).

[3]. Mark Fox, Quantum Optic s Introduction, (Oxford University publisher, 2006).

[4]. Misrak Getahun, Three Level Laser Dynamic with Coherent and Squeezed Light, PhD Depression (Addis Ababa University, 2009).

[5]. Fesseha Kassahun, Three-Level Laser Electrized Dynamics Bombardment, (Addis Ababa University, 2012).

[6]. Assegid Mengistu, MSc Thesis, (Addis Ababa University, 2010).

[7]. S. M. Barnett and P P. M. Raymore, Methods in Theoretical Quantum Optic s, (Oxford, University Press, New York, 1997)

[8]. TewodrosYirgeshewa Darge, Conducted According to Delivery Three Levels r Parametric Amplifier, PhD Dissertation (Addis Ababa University, 2010)

[9]. M. O. Scully and Mr. S. Zubairy, Quantum Optics, (Cambridge University Press, 1997).

[10]. The Wobshet Mekonen is a three-level non-abrasive laser with hole-operated methods light MSc Thesis, (Addis Ababa University, 2007).

[11]. Sintayehu Tesfa, arxiv: 07082815v1 [quanta- ph. (2007).

[12]. Sintayehu Tesfa a No degenerate Three-level Cascade Laser Couple d to a Two- mode Pressure Space Pressure (Addis Ababa University 2008)

[13]. Economou, Sophia spontaneous output and control of spins in quantum dots PhD Dissertation (UNIVE RADIT Y CARD A, 2006)

[14]. Beyene Abiti Superposed De gene rate Three-Level Lasers MSc concept (Addis Ababa University 2011)

[15]. Tizazu Maresha Superposition of Squeezed Laser Light Beams PhD Dissertation (Addis Ababa University 2015)

[16]. Solomon Getachew TWO FREE COHERENT MODES AND LIG HT BE AMS MSc PAGES concept (Addis Ababa University June 2011)

[17]. Peter Lambropoulos David Petrosyan Foundations for Quantum Optics and Quantum Information (Springer)